CS 188: Artificial Intelligence
Spring 2007

Lecture 7: CSP-II and Adversarial Search

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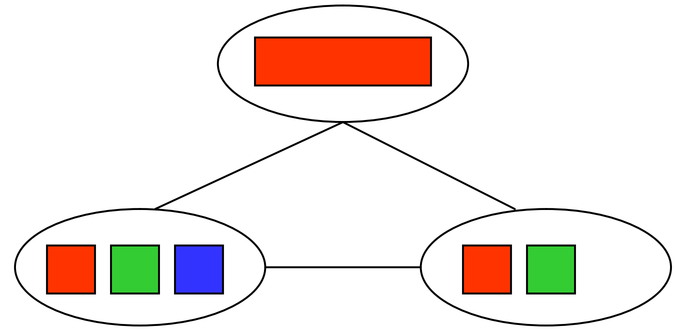
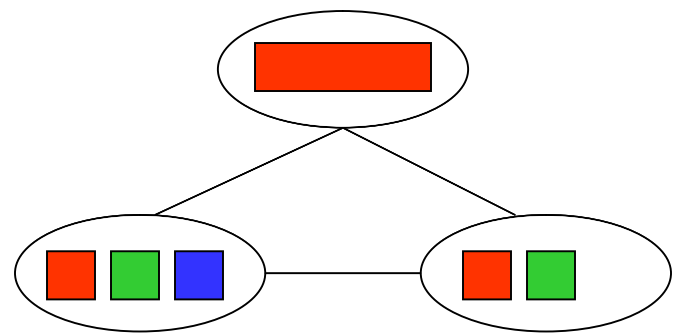
Many slides over the course adapted from Dan Klein, Stuart Russell or Andrew Moore

Summary: Consistency

- Basic solution: DFS / backtracking
  - Add a new assignment
  - Check for violations



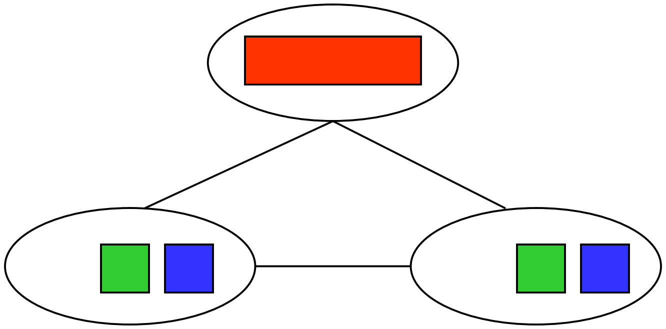
- Forward checking:
  - Pre-filter unassigned domains after every assignment
  - Only remove values which conflict with current assignments

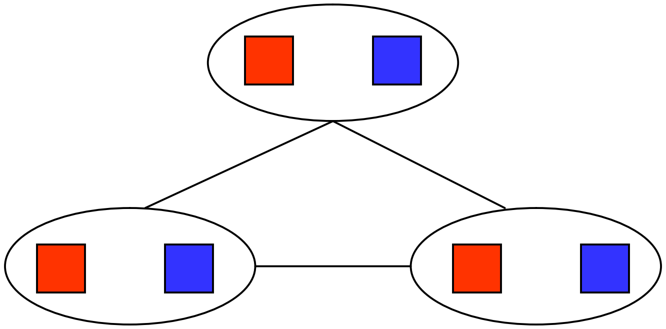


- Arc consistency
  - We only defined it for binary CSPs
  - Check for impossible values on all pairs of variables, prune them
  - Run (or not) after each assignment before recursing
  - A pre-filter, not search!

Limitations of Arc Consistency

- After running arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

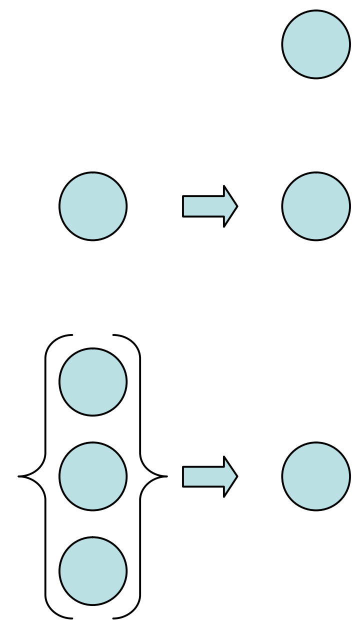




What went wrong here?

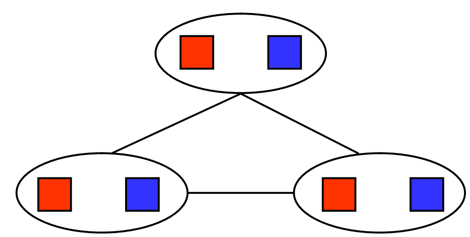
K-Consistency

- Increasing degrees of consistency
  - 1-Consistency (Node Consistency): Each single node's domain has a value which meets that node's unary constraints
  - 2-Consistency (Arc Consistency): For each pair of nodes, any consistent assignment to one can be extended to the other
  - K-Consistency: For each k nodes, any consistent assignment to k-1 can be extended to the k^{th} node.



- Higher k more expensive to compute

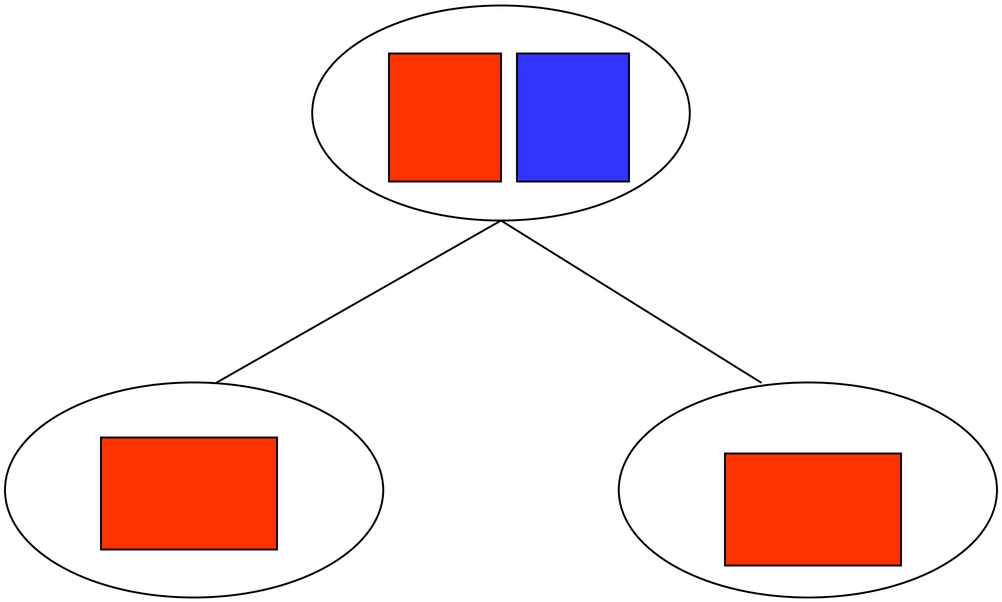
- (You need to know the k=2 algorithm)



Strong K-Consistency

- Strong k-consistency: also k-1, k-2, … 1 consistent
- Claim: strong n-consistency means we can solve without backtracking!
- Why?
  - Choose any assignment to any variable
  - Choose a new variable
  - By 2-consistency, there is a choice consistent with the first
  - Choose a new variable
  - By 3-consistency, there is a choice consistent with the first 2
  - …
- Lots of middle ground between arc consistency and n-consistency! (e.g. path consistency)

K-consistent vs. strong k-consistent



Iterative Algorithms for CSPs

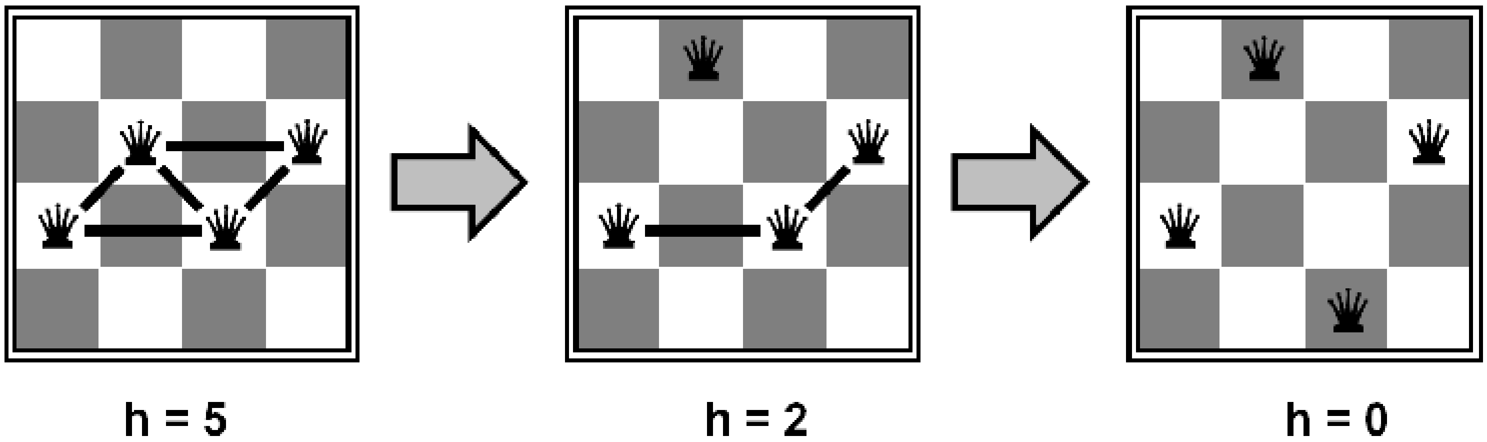
- Greedy and local methods typically work with “complete” states, i.e., all variables assigned

- To apply to CSPs:
  - Allow states with unsatisfied constraints
  - Operators *reassign* variable values

- Variable selection: randomly select any conflicted variable

- Value selection by min-conflicts heuristic:
  - Choose value that violates the fewest constraints
  - I.e., hill climb with h(n) = total number of violated constraints

Example: 4-Queens



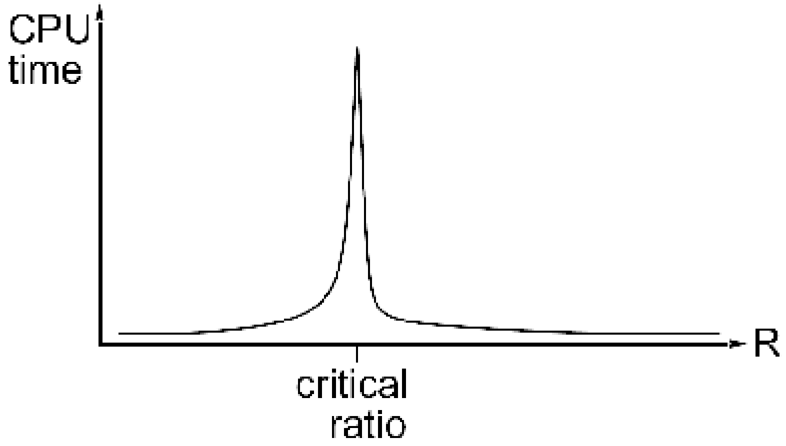
- States: 4 queens in 4 columns ($4^4 = 256$ states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: h(n) = number of attacks

Performance of Min-Conflicts

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)

- The same appears to be true for any randomly-generated CSP *except* in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



Example: Boolean Satisfiability

- Given a Boolean expression, is it satisfiable?
- Very basic problem in computer science

$$p_1 \wedge (p_2 \rightarrow p_3) \wedge ((\neg p_1 \wedge \neg p_3) \rightarrow \neg p_2) \wedge (p_1 \vee p_3)$$

- Turns out you can always express in 3-CNF

$$(p_1) \wedge (\neg p_2 \vee p_3) \wedge (p_1 \vee p_3 \vee \neg p_2) \wedge (p_1 \vee p_2 \vee p_3)$$

- 3-SAT: find a satisfying truth assignment

Example: 3-SAT

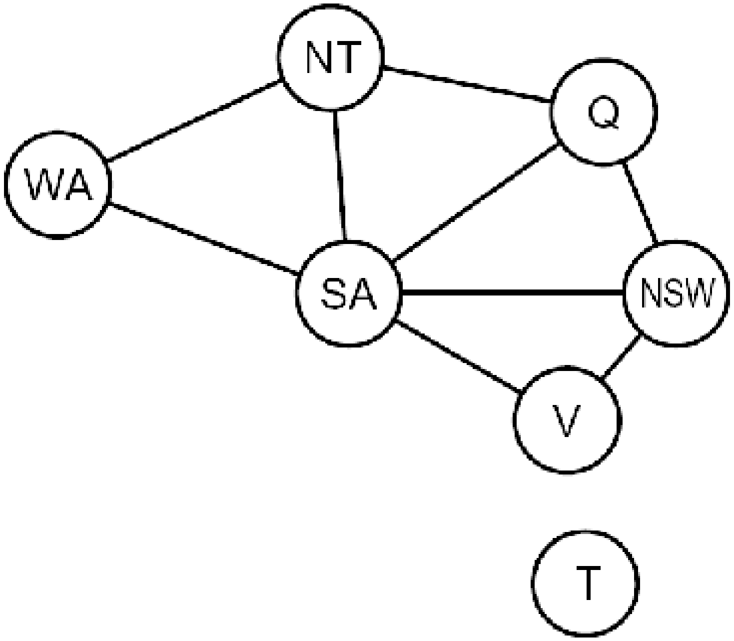
- Variables: $p_1, p_2, \cdots p_n$
- Domains: $\{\text{true}, \text{false}\}$
- Constraints:

$$\left.\begin{array}{c} p_i \vee p_j \vee p_k \\ \neg p_{i'} \vee p_{j'} \vee p_{k'} \\ \vdots \\ p_{i''} \vee \neg p_{j''} \vee \neg p_{k''} \end{array}\right\}$$

Implicitly conjoined (all clauses must be satisfied)

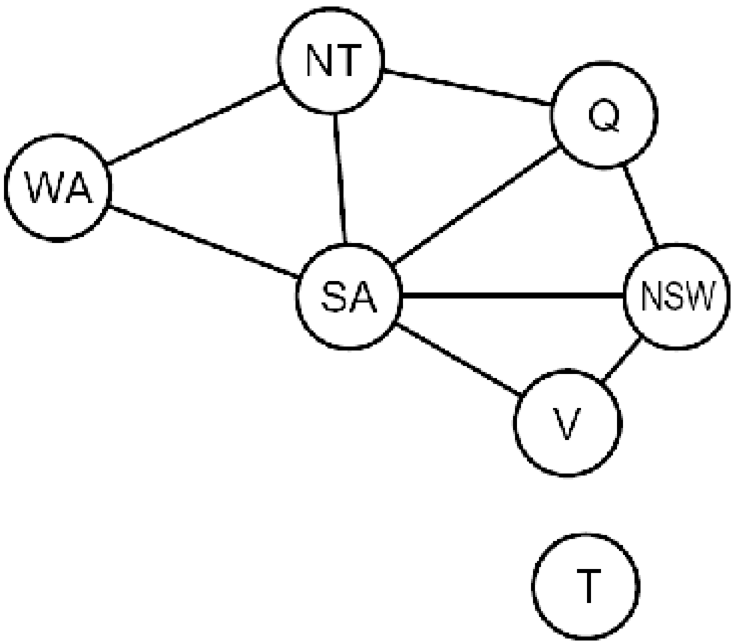
CSPs: Queries

- Types of queries:
  - Legal assignment
  - All assignments
  - Possible values of some query variable(s) given some evidence (partial assignments)

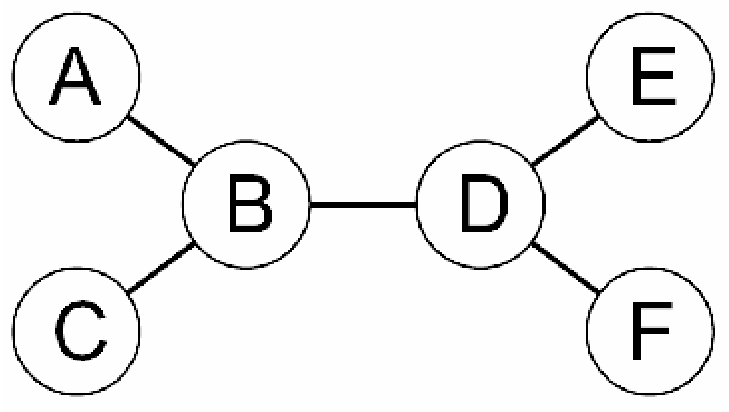


Problem Structure

- Tasmania and mainland are independent subproblems
- Identifiable as connected components of constraint graph
- Suppose each subproblem has c variables out of n total
  - Worst-case solution cost is $O((n/c)(d^c))$, linear in n
  - E.g., n = 80, d = 2, c =20
  - 2^{80} = 4 billion years at 10 million nodes/sec
  - $(4)(2^{20})$ = 0.4 seconds at 10 million nodes/sec

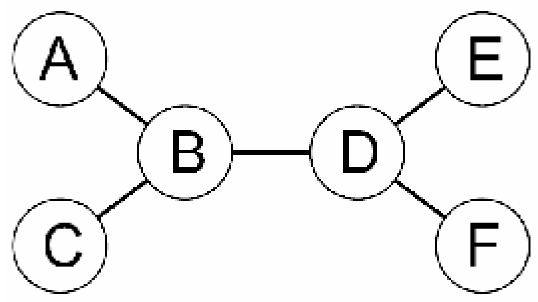


Tree-Structured CSPs

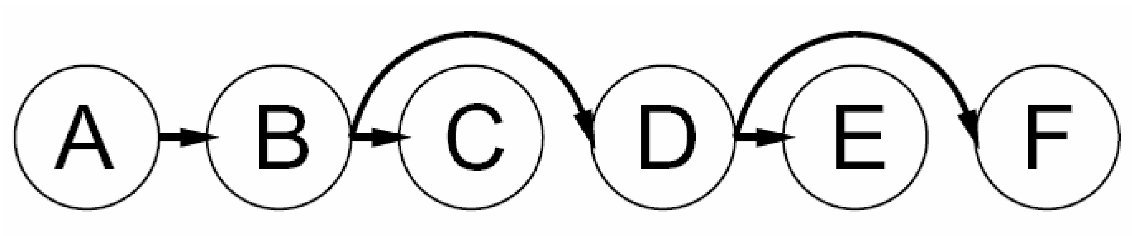


- Theorem: if the constraint graph has no loops, the CSP can be solved in $O(n\ d^2)$ time
  - Compare to general CSPs, where worst-case time is $O(d^n)$
- This property also applies to logical and probabilistic reasoning: an important example of the relation between syntactic restrictions and the complexity of reasoning.

Tree-Structured CSPs



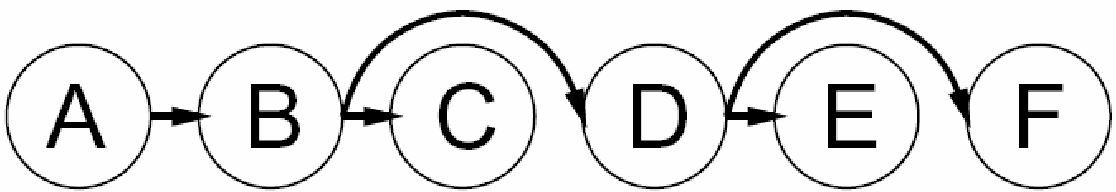
- Choose a variable as root, order variables from root to leaves such that every node's parent precedes it in the ordering



- For i = n : 2, apply RemoveInconsistent(Parent(X_i),X_i)
- For i = 1 : n, assign X_i consistently with Parent(X_i)
- Runtime: $O(n\ d^2)$ (why?)

Tree-Structured CSPs

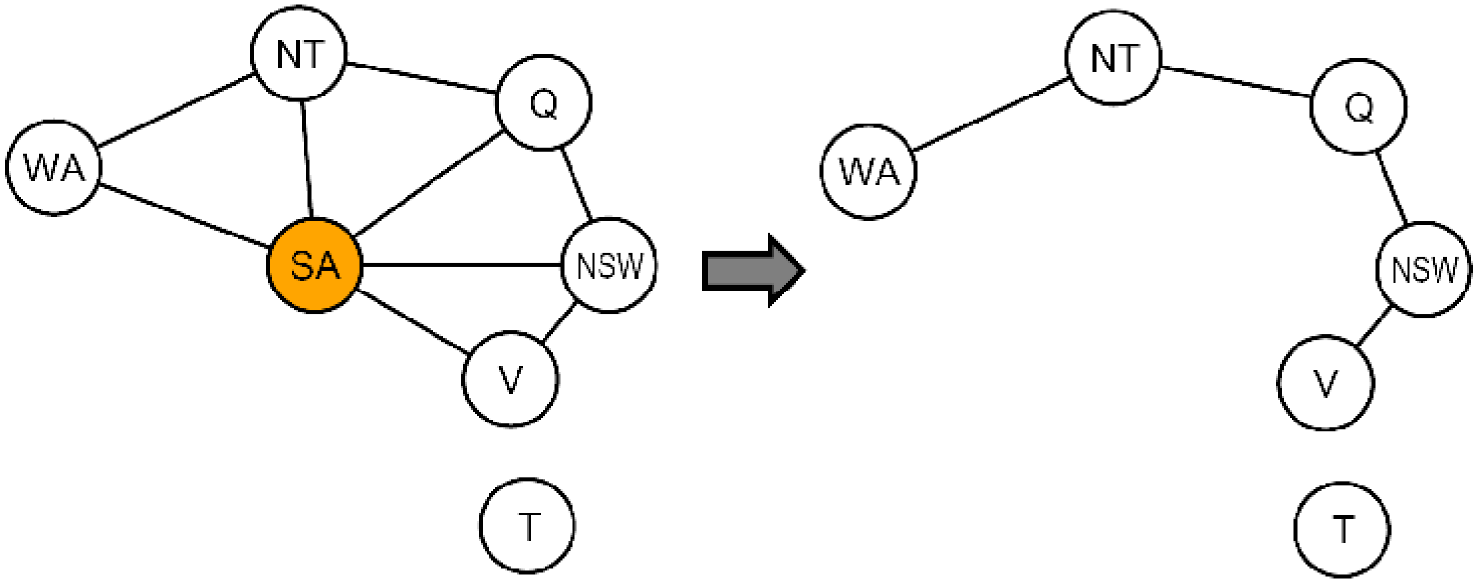
- Why does this work?
- Claim: After each node is processed leftward, all nodes to the right can be assigned in any way consistent with their parent.
- Proof: Induction on position



- Why doesn't this algorithm work with loops?

- Note: we'll see this basic idea again with Bayes' nets and call it belief propagation

Nearly Tree-Structured CSPs



- Conditioning: instantiate a variable, prune its neighbors' domains
- Cutset conditioning: instantiate (in all ways) a set of variables such that the remaining constraint graph is a tree
- Cutset size c gives runtime $O((d^c) (n\text{-}c) d^2)$, very fast for small c

CSP Summary

- CSPs are a special kind of search problem:
  - States defined by values of a fixed set of variables
  - Goal test defined by constraints on variable values
- Backtracking = depth-first search with one legal variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- The constraint graph representation allows analysis of problem structure
- Tree-structured CSPs can be solved in linear time
- Iterative min-conflicts is usually effective in practice

Games: Motivation

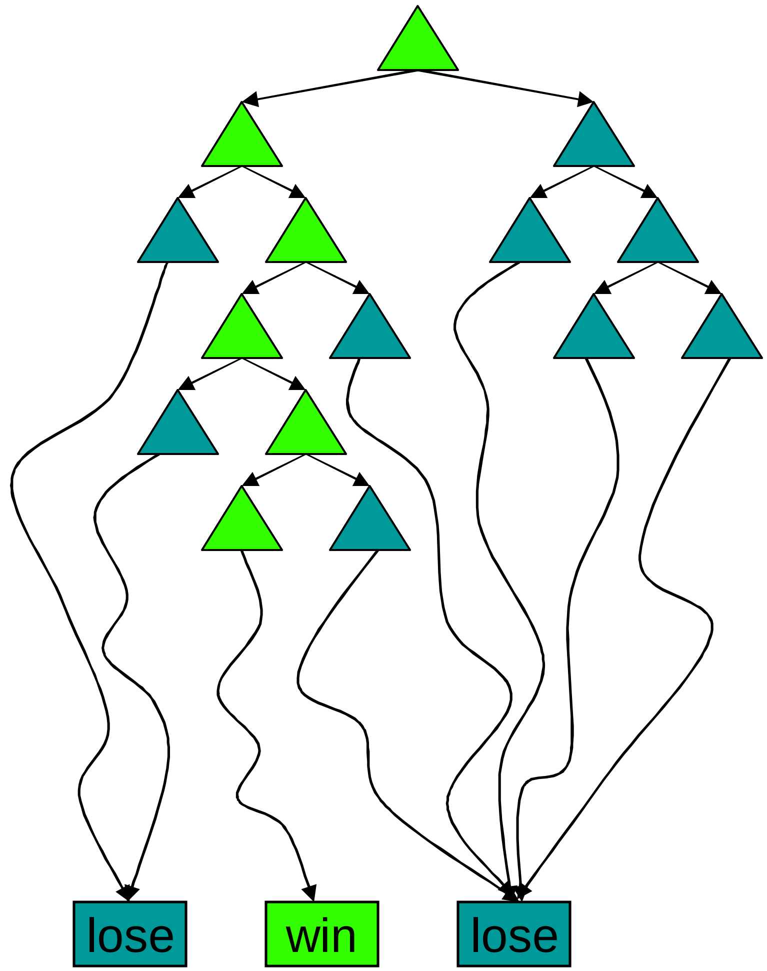
- Games are a form of *multi-agent environment*
  - What do other agents do and how do they affect our success?
  - Cooperative vs. competitive multi-agent environments.
  - Competitive multi-agent environments give rise to adversarial search a.k.a. *games*
- Why study games?
  - Games are fun!
  - Historical role in AI
  - Studying games teaches us how to deal with other agents trying to foil our plans
  - *Huge* state spaces – Games are *hard*!
  - Nice, clean environment with clear criteria for success

Game Playing

- Axes:
  - Deterministic or stochastic?
  - One, two or more players?
  - Perfect information (can you see the state)?
- Want algorithms for calculating a strategy (policy) which recommends a move in each state

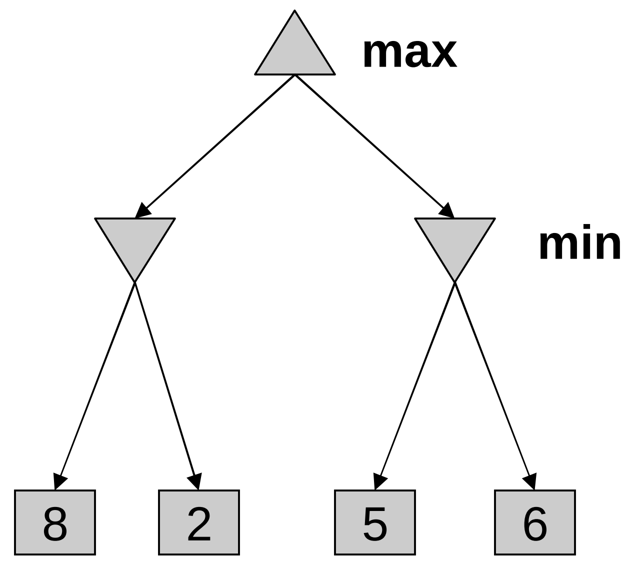
Deterministic Single-Player?

- Deterministic, single player, perfect information:
  - Know the rules
  - Know what actions do
  - Know when you win
  - E.g. Freecell, 8-Puzzle, Rubik's cube
- … it's just search!
- Slight reinterpretation:
  - Each node stores the best outcome it can reach
  - This is the maximal outcome of its children
  - Note that we don't store path sums as before
- After search, can pick move that leads to best node

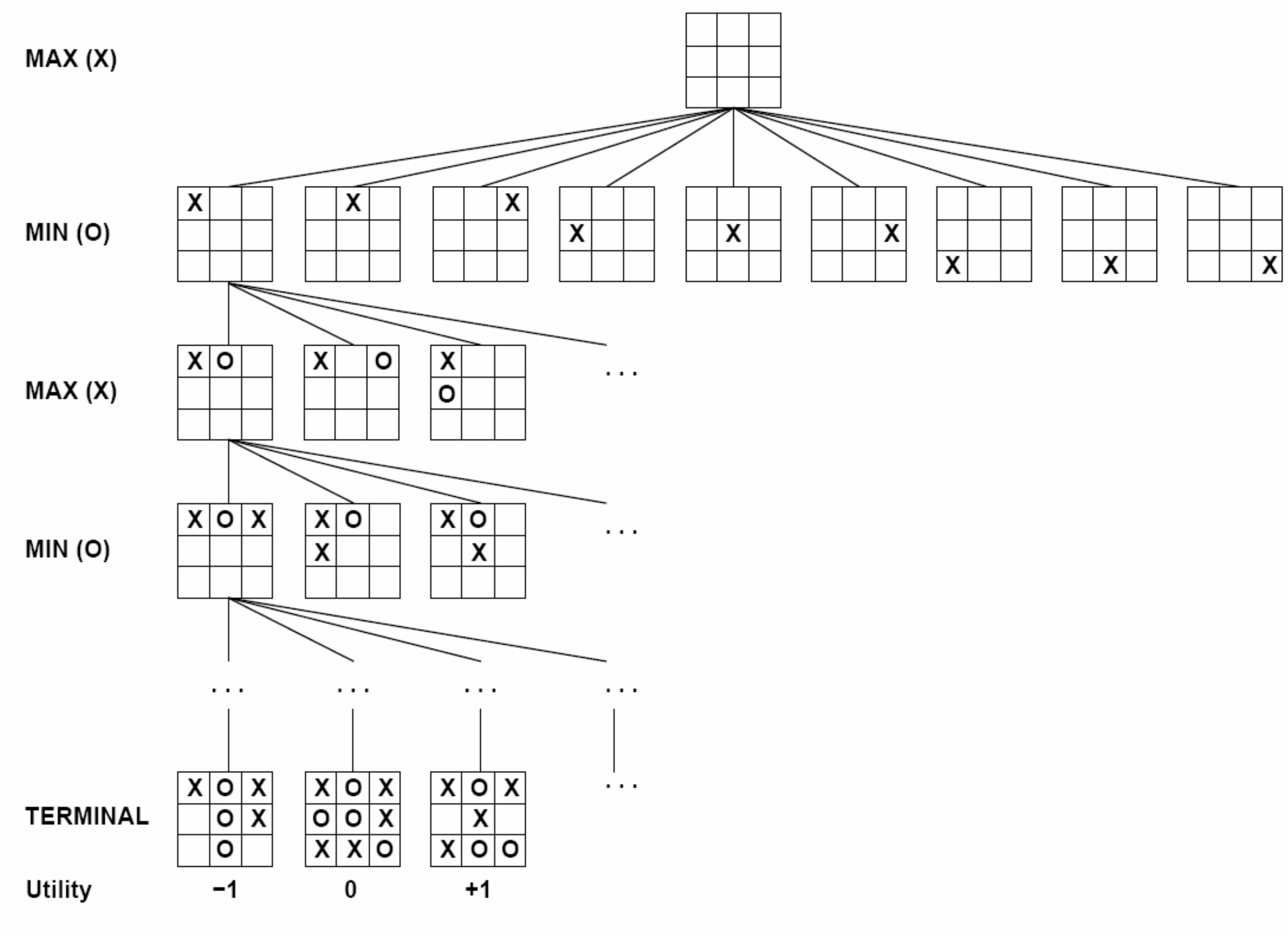


Deterministic Two-Player

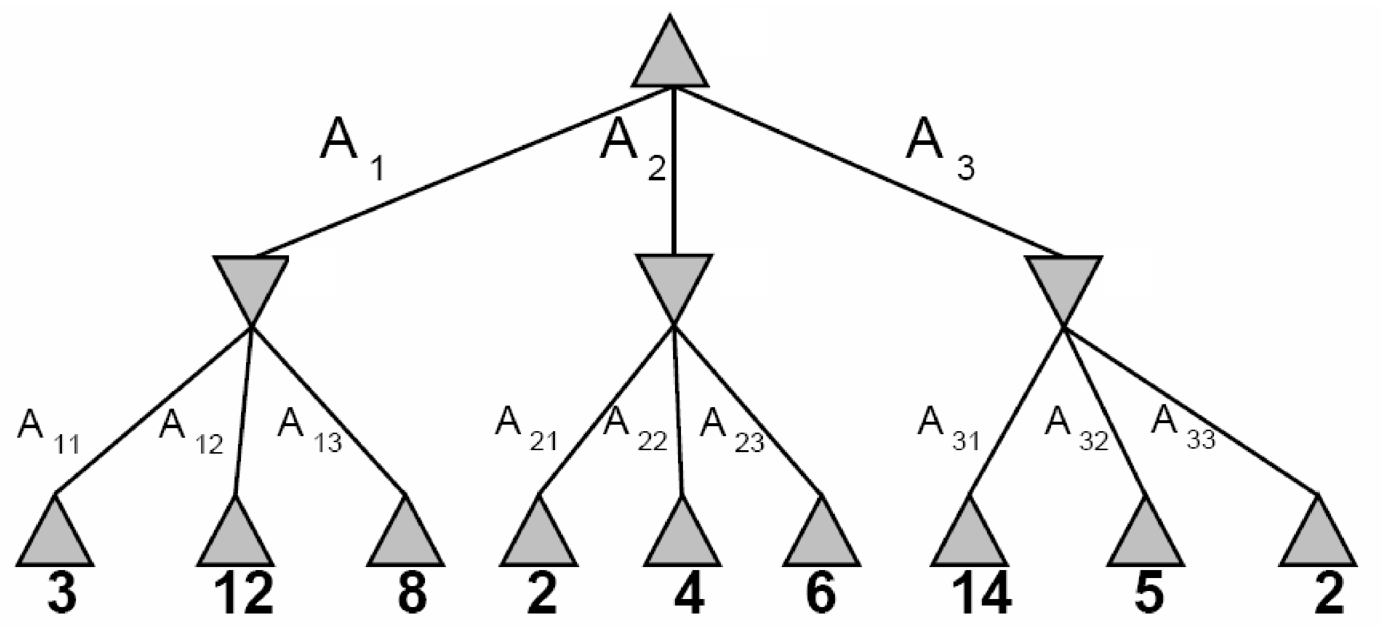
- E.g. tic-tac-toe, chess, checkers
- **Minimax search**
  - A state-space search tree
  - Players alternate
  - Each layer, or ply, consists of a round of moves
  - Choose move to position with highest minimax value = best achievable utility against best play
- Zero-sum games
  - One player maximizes result
  - The other minimizes result



Tic-tac-toe Game Tree



Minimax Example



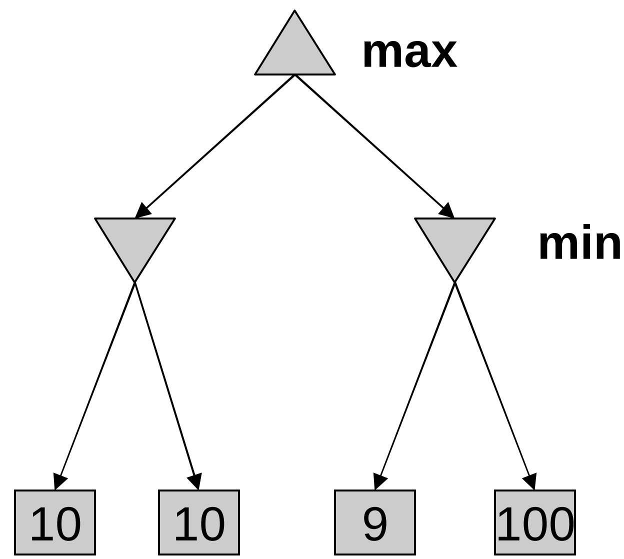
Minimax Search

```
function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do v ← MIN(v, MAX-VALUE(s))
    return v
```

Minimax Properties

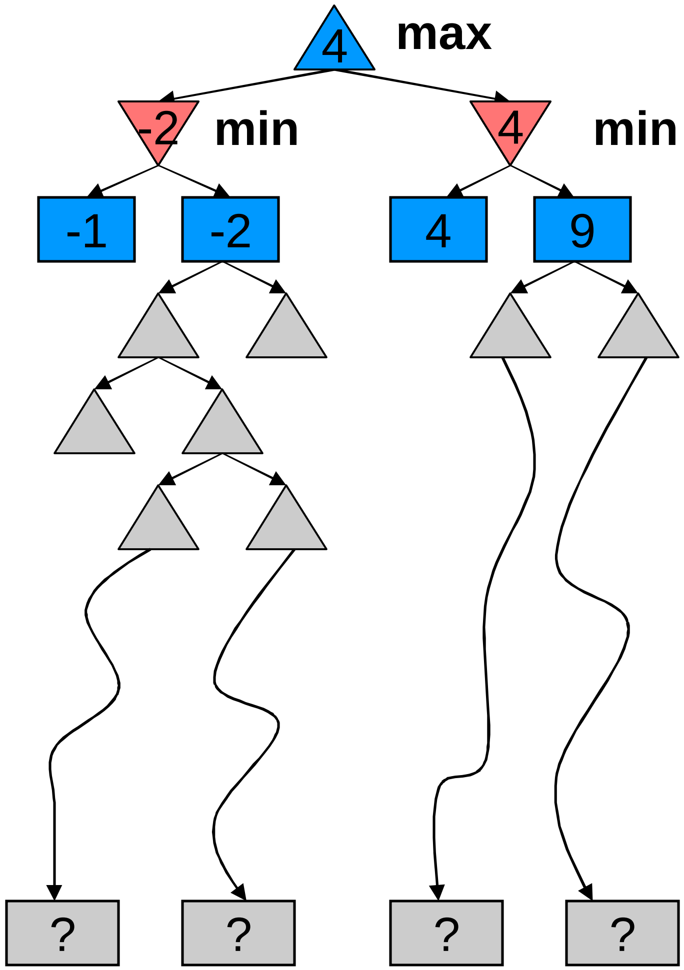
- Optimal against a perfect player. Otherwise?



- Time complexity?
  - $O(b^m)$
- Space complexity?
  - $O(bm)$
- For chess, $b \approx 35$, $m \approx 100$
  - Exact solution is completely infeasible
  - But, do we need to explore the whole tree?

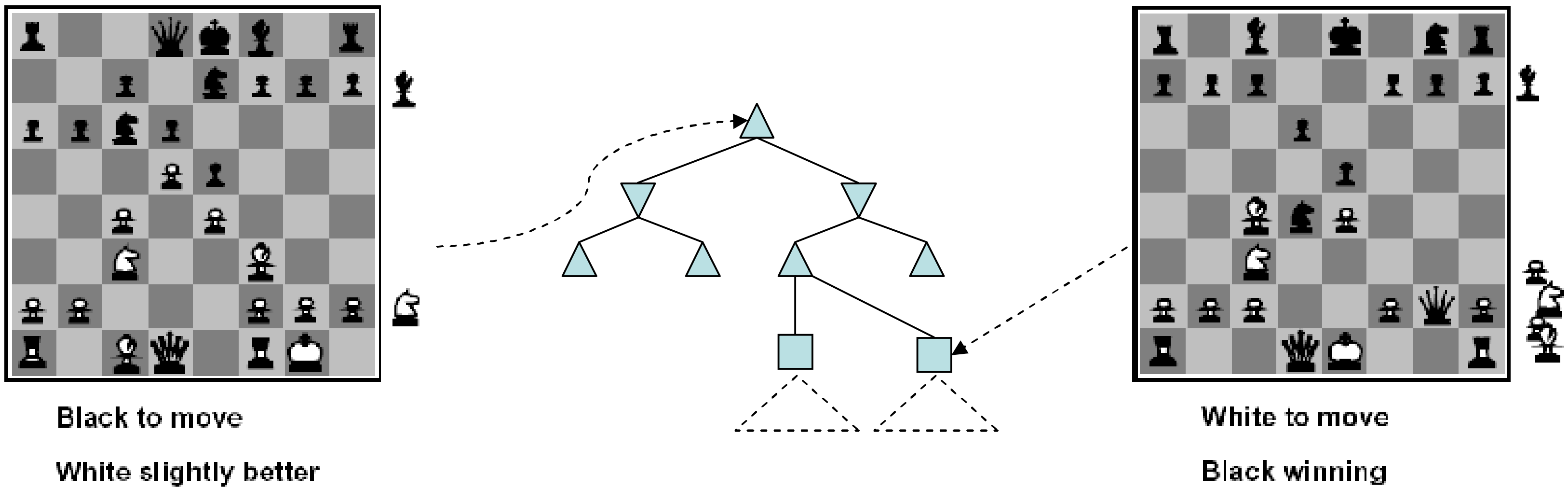
Resource Limits

- Cannot search to leaves
- Limited search
  - Instead, search a limited depth of the tree
  - Replace terminal utilities with an eval function for non-terminal positions
- Guarantee of optimal play is gone
- More plies makes a BIG difference
- Example:
  - Suppose we have 100 seconds, can explore 10K nodes / sec
  - So can check 1M nodes per move
  - α-β reaches about depth 8 – decent chess program



Evaluation Functions

- Function which scores non-terminals



- Ideal function: returns the utility of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

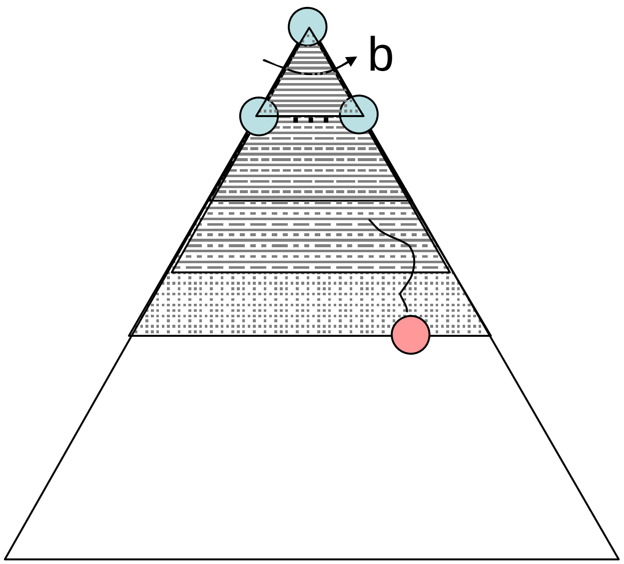
- e.g. $f_1(s)$ = (num white queens – num black queens), etc.

Iterative Deepening

Iterative deepening uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less. (DFS gives up on any path of length 2)
2. If “1” failed, do a DFS which only searches paths of length 2 or less.
3. If “2” failed, do a DFS which only searches paths of length 3 or less.

….and so on.

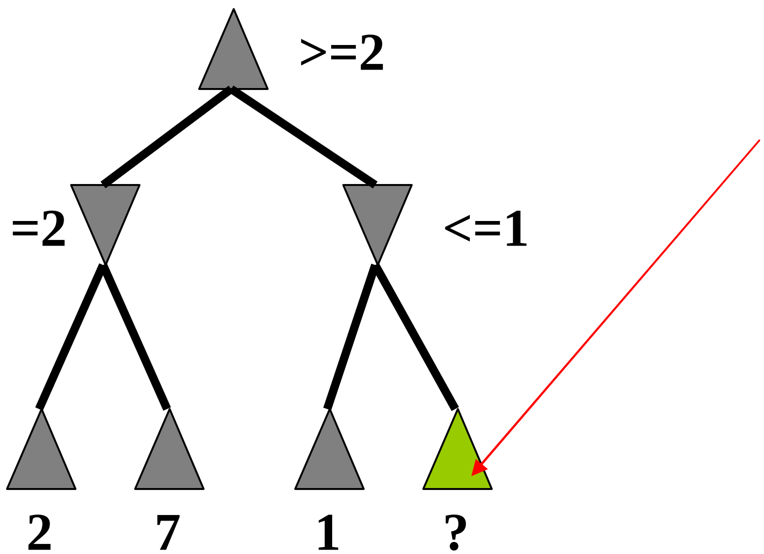


This works for single-agent search as well!

Why do we want to do this for multiplayer games?

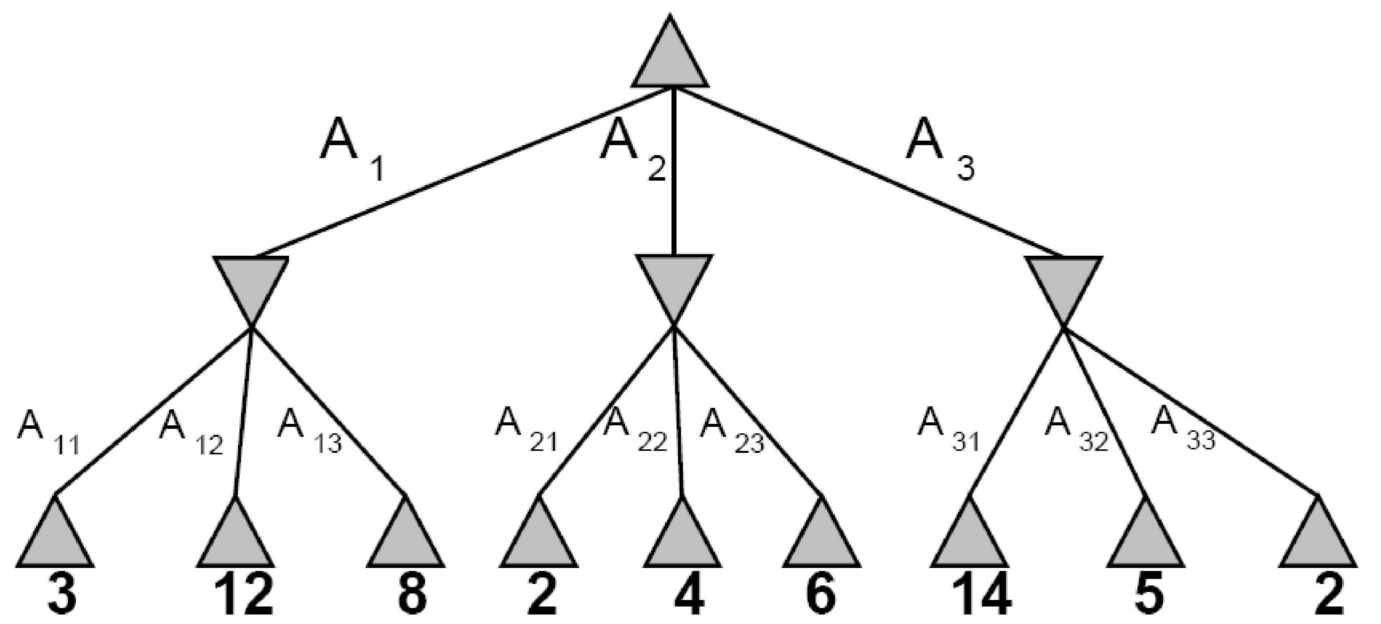
Alpha-Beta Pruning

- A way to improve the performance of the Minimax Procedure
- Basic idea: *"If you have an idea which is surely bad, don't take the time to see how truly awful it is"* ~ Pat Winston

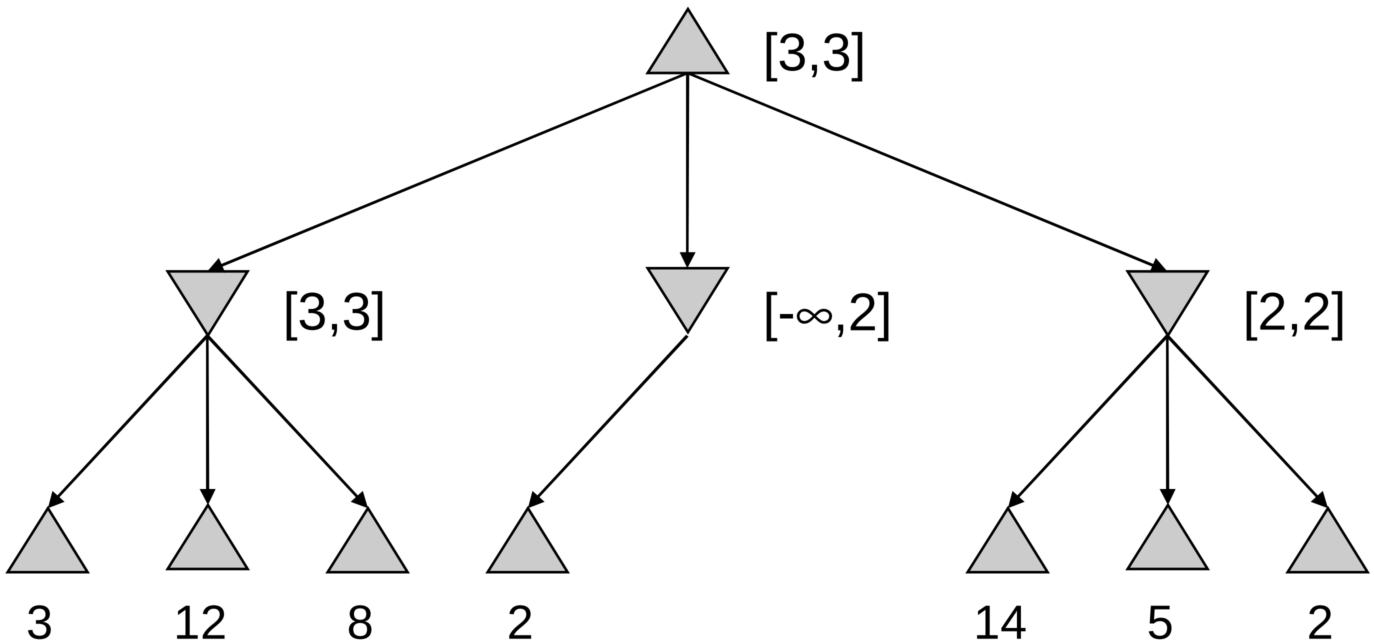


- We don't need to compute the value at this node.
- No matter what it is it can't effect the value of the root node.

α-β Pruning Example

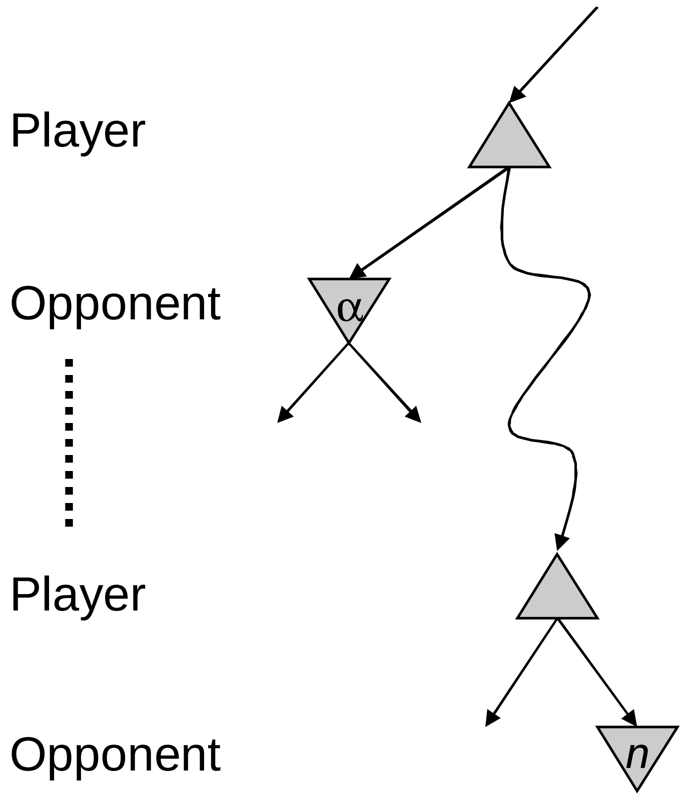


Pruning in Minimax Search



α-β Pruning

- General configuration
  - α is the best value the Player can get at any choice point along the current path
  - If *n* is worse than α, MAX will avoid it, so prune *n*'s branch
  - Define β similarly for MIN

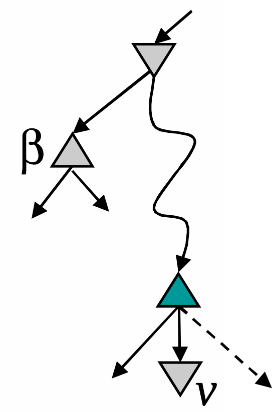


α-β Pruning Pseudocode

```
function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
    return v
```

```
function MAX-VALUE(state, α, β) returns a utility value
    inputs: state, current state in game
            α, the value of the best alternative for  MAX along the path to state
            β, the value of the best alternative for  MIN along the path to state

    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s, α, β))
        if v ≥ β then return v
        α ← MAX(α, v)
    return v
```



α-β Pruning Properties

- Pruning has no effect on final result

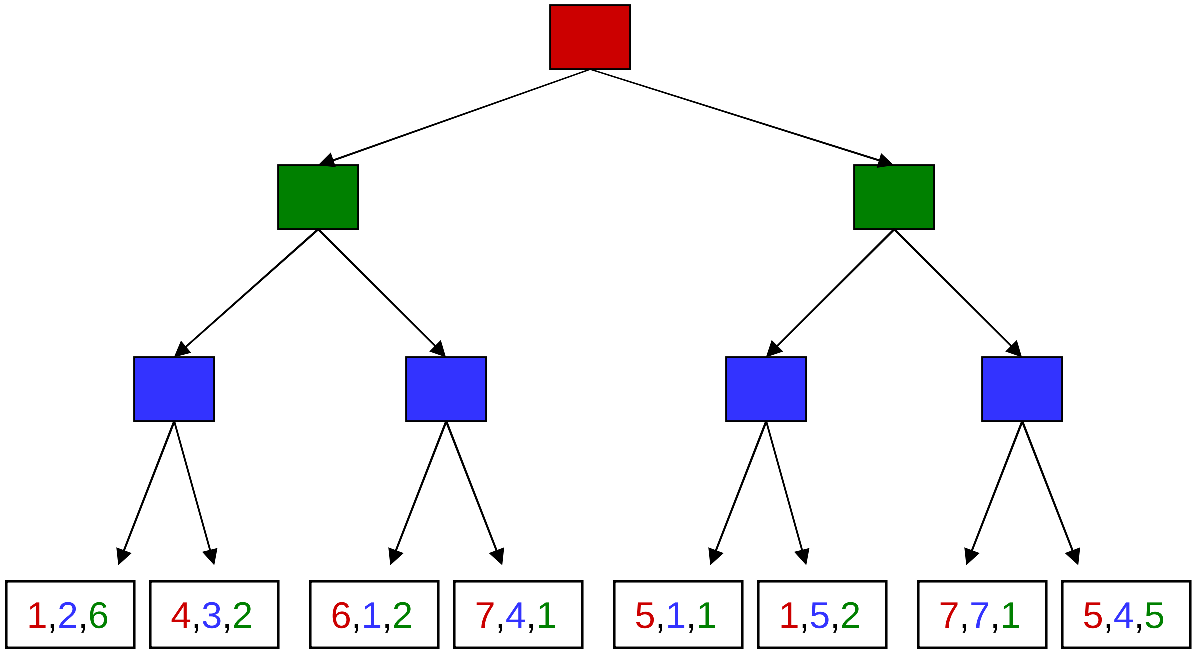
- Good move ordering improves effectiveness of pruning

- With "perfect ordering":
  - Time complexity drops to $O(b^{m/2})$
  - Doubles solvable depth
  - Full search of, e.g. chess, is still hopeless!

- A simple example of metareasoning, here reasoning about which computations are relevant

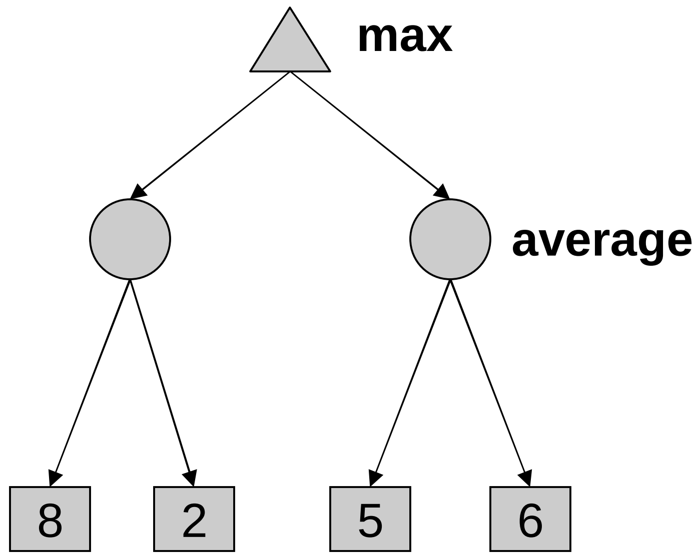
Non-Zero-Sum Games

- Similar to minimax:
  - Utilities are now tuples
  - Each player maximizes their own entry at each node
  - Propagate (or back up) nodes from children



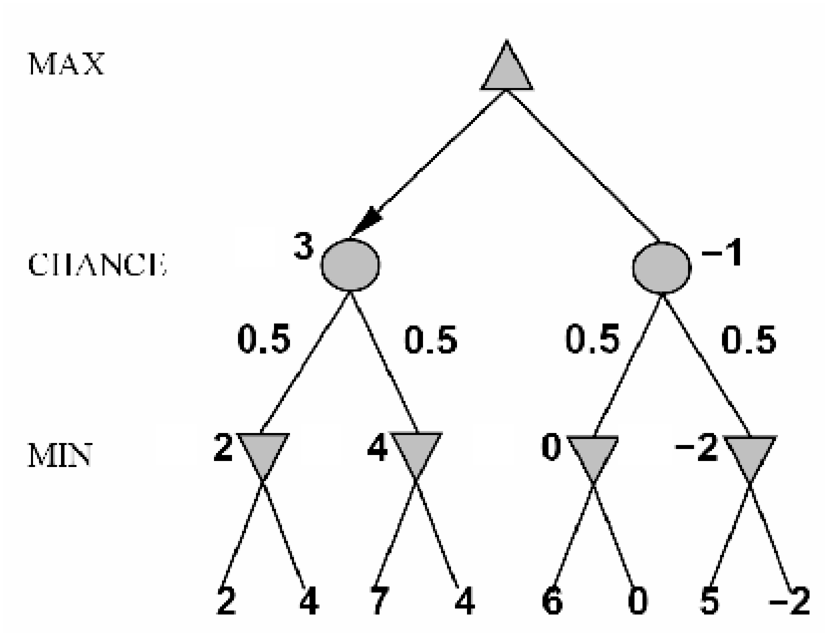
Stochastic Single-Player

- What if we don't know what the result of an action will be? E.g.,
  - In solitaire, shuffle is unknown
  - In minesweeper, don't know where the mines are
- Can do **expectimax search**
  - Chance nodes, like actions except the environment controls the action chosen
  - Calculate utility for each node
  - Max nodes as in search
  - Chance nodes take average (expectation) of value of children
- Later, we'll learn how to formalize this as a **Markov Decision Process**



Stochastic Two-Player

- E.g. backgammon
- Expectiminimax (!)
  - Environment is an extra player that moves after each agent
  - Chance nodes take expectations, otherwise like minimax



if *state* is a MAX node **then**
  return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a MIN node **then**
  return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
if *state* is a chance node **then**
  return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

Game Playing State-of-the-Art

- **Checkers:** Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

- **Chess:** Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue examined 200 million positions per second, used very sophisticated evaluation and undisclosed methods for extending some lines of search up to 40 ply.

- **Othello:** human champions refuse to compete against computers, which are too good.

- **Go:** human champions refuse to compete against computers, which are too bad. In go, b > 300, so most programs use pattern knowledge bases to suggest plausible moves.

Stochastic Two-Player

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 4 = 20 x (21 x 20)^3 1.2 x 10^9
- As depth increases, probability of reaching a given node shrinks
  - So value of lookahead is diminished
  - So limiting depth is less damaging
  - But pruning is less possible…
- TDGammon uses depth-2 search + very good eval function + reinforcement learning: world-champion level play